

$$\sum_{k=0}^n k^2 = \square + \square + \square + \square + \square + \dots + \square =$$

$$= \left\{ \begin{array}{ccccccc} \square & \square & \square & \square & \dots & \square \\ \square & \square & \square & \square & \dots & \square \\ \square & \square & \square & \square & \dots & \square \\ \square & \square & \square & \square & \dots & \square \\ \end{array} \right\} =$$

$$= \sum_{k=0}^n (n-k)(2k+1)$$

$$\sum_{k=0}^n k^2 = \sum_{k=0}^n (2kn - 2k^2 + n - k)$$

$$\sum_{k=0}^n k^2 = 2n \sum_{k=0}^n k - 2 \sum_{k=0}^n k^2 + (n+1)n - \sum_{k=0}^n k$$

$$3 \sum_{k=0}^n k^2 = 2n \sum_{k=0}^n k + (n+1)n - \sum_{k=0}^n k$$

$$3 \sum_{k=0}^n k^2 = (2n-1) \frac{(n+1)n}{2} + (n+1)n$$

$$3 \sum_{k=0}^n k^2 = \frac{(2n+1)(n+1)n}{2}$$

$$\sum_{k=0}^n k^2 = \frac{(2n+1)(n+1)n}{6}$$